LOGARITHMIC RISK DISTRIBUTION TO BUILD A STABLE CAPITAL GROWTH FOR ANY BUSINESS OR INVESTMENT

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ABSTRACT

Capital investment, trading, or any business, all are activities involving risk. A proper capital management strategy to ensure long-term profitability is valuable nowadays. High price volatility in the stock markets, unpredicted or unusual economic and geopolitical news, or just hard to manage rare resources or human factors, all of these are instability reasons which can decrease capital efficiency or even cancel the profit over time. To manage the involved risk in any economic activity is a key factor for any manager today. Whatever the risk is estimated, the basic idea is always the same. To ensure long-term profitability, the investor has to save a part of the profit and to reinvest the rest to obtain a stable capital grow in time. The question this paper will answer is how much profit to save and how much to reinvest to produce stable capital growth and sustainable capital efficiency? The Logarithmic Risk Distribution will be presented, a practical method to size the risk level depending on the invested capital, on the used capital exposure level, and on the profit already made in the current business. It was found that the risk level can depend only on these three factors through a function that will provide an exponential capital grow even if the risk is higher than the realized profit. This paper will also include examples to prove the efficiency and simplicity of the presented method. The Logarithmic Risk Distribution is simple and easy to be applied in any business or investment.

KEYWORDS: *business management, capital investment, logarithmic risk distribution, trading.*

1. INTRODUCTION

To make a profit is not enough today. For a sustainable activity, for long-run business development, any manager has to provide long-term profitability. It is well known that any capital investment, any trading activity, and in fact, any business is involving a risk. High price volatility in the stock markets, financial crisis, unpredicted economic or geopolitical news, or only hard to manage rare resources or human factors, all of these are instability reasons which can decrease capital efficiency or even can cancel the business profit over a significant period of time. No matter how the risk is measured, a proper capital and risk management strategy can ensure stable business growth in time. The problem analyzed in this paper is how much to save from the obtained profit and how much to reinvest in order to build a stable profit growth over time. Sustained capital management is a crucial factor in any economic activity. A common problem for any manager today is to find the proper profit and risk distribution. The methods of how to measure the risk involved in the current economic activity are not a subject of this paper. In this paper, we will consider those methods to be known, and the risk can be accurately established at any moment in our considered activity.

In those activities where the profit is higher than the involved risk, the risk distribution is easy to be set by empirical or experimental models. Formulas, as to save half of the profit or save two-thirds or

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three quarters and reinvest the rest, are well tested before for hundreds of years by our predecessors. These cases will not be a subject of this paper. A more important problem is emitted for those cases when the profit level is lower than the involved risk in a specified period of time. It is well known that in tremendously higher activity domains, the risk involved today is much higher than the profit obtained. This fact is not because of a lack of experience, or because of inadequate capital management, but due to the specificity of the considered domain. Capital investment on shares, short-time financial trading, real estate investments, huge allocated resources businesses, are only a few examples of cases in which the risk distribution becomes more significant today. A substantial loss in these cases can delete the profit for an extended period of time, and can even conduct to bankruptcy in that activity. The risk distribution presented in this article is designed especially for these cases when the risk level is higher than the recorded profit in a specified period.

This paper will present the Logarithmic Risk Distribution to answer to the enounced problem. It was found that, in any business case, the current risk level can depend only on the initial invested capital, on the capital exposure level used, and on the current profit obtained in the economic activity. The Logarithmic Risk Distribution will provide the risk percentage for each moment of time in order to produce an exponential capital growth for the next period. The specificity of the business is not essential; the presented risk distribution can be applied for any economic activity, as we will see in this article. The presented capital management formula was specially designed for those cases when the risk involved is higher than the recorded profit. The presented distribution can also be used for the cases when the profit is higher than the risk, as we can see later in the paper.

The Logarithmic Risk Distribution presented in this article is a straightforward money management strategy. It can be adapted to any economic activity once the risk can be estimated. Anyone can compute the presented method with small resources in order to manage any economic activity involving a capital risk. The model permits capital and risk adjustment over time in order to ensure the adaptability for different cases or situations. The Logarithmic Risk Distribution can be applied in any capital investment to manage any business or trading activity. It can be used even to build a stable gambling money management strategy. This paper will include some examples to prove the model's simplicity and efficacy.

2. BIBLIOGRAPHY REVIEW

As it was mentioned before, the problem discussed in this paper is the profit distribution. Considering we have a stable and profitable economic activity, involving a measurable risk level, we ask how much from the realized profit we have to save, and how much we have to reinvest in the current activity, in order to build a sustained capital growth over the time. The money management and risk aversion problems are a high-interest subject for hundreds of years. There are well known Bernoulli's empirical solutions for the risk assessment and risk distribution (Eeckhoudt, Gollier & Schlesinger, 2005) even from the eighteenth century. Splitting the income or the profit in more parts with different risk allocation are methods tested before by our predecessors. Save half, save two-thirds or save three-quarters and reinvest the rest are empirical methods for we have not a specific author. These are functional models, especially when the income or profit is much higher than the risk. When the risk is higher than the income, these models are failing when a significant loss is met. For these cases, we need a stable methodology for the risk distribution.

Over time more money management strategies and risk distribution functions or methods have been imagined depending on the specificity of each activity. Different working models and original approach for money management strategies can be found in the literature. One of the most active domains that substantially contribute to the money management strategy development is the capital trading and investment domain. The high-risk involved in this field imposes significant attention on the money management steps in order to conserve the invested capital. Over time more significant methodologies for the money and risk management were developed. Especially for capital investment activity, substantial considerations about money and risk management strategies can be found in (Dormeier, 2011), (Farley, 2010), (Chan, 2009), (Bland, Meisler & Archer, 2009), (Dologa, 2008), (Corcoran, 2007), (Mac, 2006), (Mun, 2006), (Weissman, 2005), (Elder, 2002), (Connors & Radchke, 1995), and (Elder, 1993). Because taking the risk has a significant psychological impact factor, and because individuals may have different risk attitudes (Hillson & Murray-Webster, 2007), the risk assertion and the risk aversion are factors that must be taken into consideration on a general risk distribution. From the psychological point of view, several authors have a substantial contribution as (Ward, 2009), (Kiev, 2008), or (Pring, 1992).

The second one domain participating significantly in the risk and capital management strategy development is the corporate finance domain, in which the profit distribution is usually made depending on the specificity of the current activity. Significant approaches and models can be found in (Basak & Makarov, 2014), (Saks & Maringer, 2009), (Armstrong & Murlis, 2007), (Basak, Pavlova & Shapiro, 2007), (Power, 2004), (Stultz, 1996), or (Vince, 1992). None of these strategies are generally applicable. Some of them are not functioning when the risk is higher than the realized profit. This is a gap that will be filled by this paper.

3. WORKING HYPOTHESYS

In this paper, we want to present a risk distribution, a mathematical formula defining the risk level, which will be imposed by the manager on each step of the economic activity involved. In this chapter, we will define the hypotheses we will use to build this function, and the variables used.

As we already affirmed, comparing the risk and the profit level, the economic activities can be divided into two major parts. The first part includes those activities in which the considered risk is lower than the profit made into a specified time. An interesting example related to the capital trading activity is one of using an investment strategy with the risk and reward ratio lower than one. In this case, the profit is higher than the assumed risk. For each one dollar risked, the strategy will produce more than one dollar profit. This case is a happy one when a loss will preserve the initial capital, once the number of the winning trades is higher than the number of the losing trades on a specified period. Such as strategies are rare, and can be implemented in limited market conditions.

In the majority of cases, the risk is much higher than the realized profit. This is our first hypothesis. This case is the real one, the situation of the most capital investment strategies, the majority of any trading activity, the case of all real-estate investments, and the situation in almost all businesses in the first step of implementation. Usual to set a business, initial capital is involved, an initial amount of resources is set in order to organize the business activity, to hire the first employees, and to produce the first profit stakes. It is well known that any important investment will be covered by the realized profit in the next years, meaning that the realized profit per year is much lower than the capital involved and the resources used.

The second hypothesis is related to the phases in which the risk level is changed or not. We will consider our economic activity is carried out through two different phases. In the first one, the implementation phase, the initial risk involved is kept unchanged until a significant profit is made. In the second phase, the risk will be changed by the manager, through the risk distribution function we build, depending on the activity results, in order to improve and to increase the capitalization. We will call this second phase to be the maturity phase of our business.

In the implementation phase, an initial capital (C) is involved, an initial risked capital ($R = r \cdot C$) is allocated in order to realize an initial profit stake ($P = p \cdot C$). The measure r is the risk rate, and p is the profit rate, in order to define all the measure depending on the initial capital. By hypothesis, until P is not achieved, the risk R remains unchanged. In the implementation phase, the safe capital (S) is defined by the difference between the initial capital and the risked capital:

$$S = C - R = C \cdot (1 - r) \qquad (1)$$

On a simple example, an initial capital *C = 1,000,000 $* is invested in an economic activity with a risk rate of $r = 10\%$. The risked capital is $r = 100{,}000\$$ and the safe capital $s = 900{,}000\$$. In the implementation phase, by example, a profit rate $p = 30\%$ of the initial capital will be obtained at some point in time. To avoid confusion, we have to mention that we are still in the case in which the profit is lower than the risk involved in a specified period of time. In the current example, the profit rate *p* mentioned above defines the implementation phase. This phase can include several years of activity, in each year, the realized profit being lower than the risk rate *r*. For example, is the profit rate per year is 5%, the implementation phase in our example will include 6 years of activity. The cases when the profit rate is higher than the profit rate per year can be easily included in the same algorithm. Anyway, by the initial business plan, in this period, the capital risk rate *r* is set as fixed, as we have presumed. At the end of the implementation phase, once the profit *P* is achieved, the total available capital will be given by:

$$C_1 = C + P = C \cdot (1 + p) \qquad (2)$$

The third hypothesis in our model is related to the maturity phase. In this phase, the initial business plan was achieved; the activity proved that it could produce a profit rate p with a capital risk rate r in a specified period of time. Once the realized profit in the implementation phase is much higher than the risk involved, even the yearly profit rate is lower than the risk rate, from this moment, it can be considered that the risk can be increased in order to improve the results for the next periods and to speed up the business.

A common mistake in the second phase is to multiply the risk rate with a high number. Even a new risk rate equal $2 \cdot r$ for the second period is a bad idea; it can cancel the profit for four years of activity in our example if a significant loss is recorded. In this case, the capital will decrease significantly, a longer period of time being required to recover that loss. A better idea is to increase the risk with a very small value. The new risk level will be close to the initial risk level included in the initial business plan, a more stable functionality of all economic processes being expected.

Also, by hypothesis, the maturity period will be divided into small time intervals. On each time interval, the risk will be increased by a small δr risk gradient. In time, each interval will be defined until a small profit δp is obtained. When δr and δp are small, the number of the time intervals into the maturity period of our business is high. When these measures tend towards zero, we want to know a distribution function to define the risk level at any time moment. This is actually the problem of the manager in the maturity interval: to know how much to set the risk level. The answer will be logically deducted in the next chapter.

4. LOGARITHMIC RISK DISTRIBUTION

As was mentioned before, the maturity period is divided into small time intervals defined in time by achieving the δp profit. For each time interval the risk rate is increased with δr. Noting with (*i*) the time interval index, we can write the general formula for the considered risk rate for each interval:

$$r_i = r + i \cdot \delta r \text{ for } i \geq 1 \qquad (3)$$

By definition, each time interval last until a δp profit rate is achieved, δp being applied to the initial capital for each time interval. In this case, we will have the capital at the end of each interval defined by:

$$C_i = C_{i-1} + \delta r \cdot C_{i-1} = C_{i-1}(1+\delta p) \text{ for } i \geq 2 \qquad (4)$$

where C_1 is defined by the formula (2). Solving the recurrent formula (4), it can be simplified in:

$$C_i = C(1+p)(1+\delta p)^{(i-1)} \text{ for } i \geq 1 \qquad (5)$$

In formula (5), *C*, *p*, and δp are known terms. This fact permits to have a relation between the current capital (which can be measured by the manager at anytime) and the risk involved in any time interval in the maturity phase of the economic activity. Using the natural logarithm function and some simple transformations, we can find the (*i*) interval for each capital amount given by:

$$i = 1 + \ln\left(\frac{C_i}{C(1+p)(1+\delta p)}\right) \qquad (6)$$

In this way, using formula (3), we will find the Logarithmic Risk Distribution which will define the risk associated with each capital level:

$$r_i = r + \delta r + \ln\left(\frac{C_i}{C(1+p)(1+\delta p)}\right)^{\delta r} \qquad (7)$$

Formula (7) is still hard to be applied; the current risk level r_i is expressed as a function of the capital C_i which is the capital at the end of the time interval. Usually, the manager can measure the capital at the beginning of each time interval. Using formula (4), we can obtain a match more convenient form of the Logarithmic Risk Distribution:

$$r_i = r + \delta r + \ln\left(\frac{C_{i-1}}{C(1+p)}\right)^{\delta r} \qquad (8)$$

in which the term δp is completely excluded. In practice, it is more convenient to work with δp instead of δr, once the time interval is defined until δp is achieved. Usually, there is a direct correlation between these two terms. The manager can set, for example, to increase the risk with 1% each time when the capital was increased with 10%. This is only an example. To generalize for any other numbers, we can have a direct linear dependence defined by:

$$\delta r = \xi \cdot \delta p \qquad (9)$$

In this particular case, the Logarithmic Risk Distribution will be defined by:

$$r_i = r + \xi \cdot \delta p + \xi \cdot \ln\left(\frac{C_{i-1}}{C(1+p)}\right)^{\delta p} \qquad (10)$$

The risk r_i for each time interval will be modified depending on the initial capital *C*, on the initial risk rate *r* used to make the initial profit *p*, all of these terms being established by the initial business plan. The term ξ, the relation between the risk increment and the realized profit is defined by the

business plan set for the maturity period of our activity. Having all of these, each capital amount, which is perfectly measurable, will define the proper risk level by formula (10).
The most important advantage introduced by the Logarithmic Risk Distribution is the fact that the capital is evolving in the maturity phase through an exponential function. This fact is revealed by applying the exponential function to all terms of the relation (7). After some common operations, we will obtain:

$$C_i = C(1+p)(1+\delta p)e^{\left(\frac{r_i-(r+\delta r)}{\delta r}\right)} \qquad (11)$$

relation in whom the terms in front of the exponential function are constant once the business plan is set. Considering relation (3) the term r_i can be excluded and we will obtain much simpler:

$$C_i = C(1+p)(1+\delta p)e^{(i-1)} \text{ for } i \geq 1 \qquad (12)$$

formula confirmed by the first term. For i=1 we have the capital at the end of the first time interval in the maturity phase defined by $C_1 = C(1+p)(1+\delta p)$, in perfect accordance with our model and all considered hypothesis.

5. RESULTS

In this chapter, we will see the results of applying the Logarithmic Risk Distribution in real cases. The first example is the one we have already considered. In figure 1, is presented the capital evolution in time. It can be observer the constant evolution in the implementation phase and the exponential evolution realized in the maturity phase because of the risk increase.

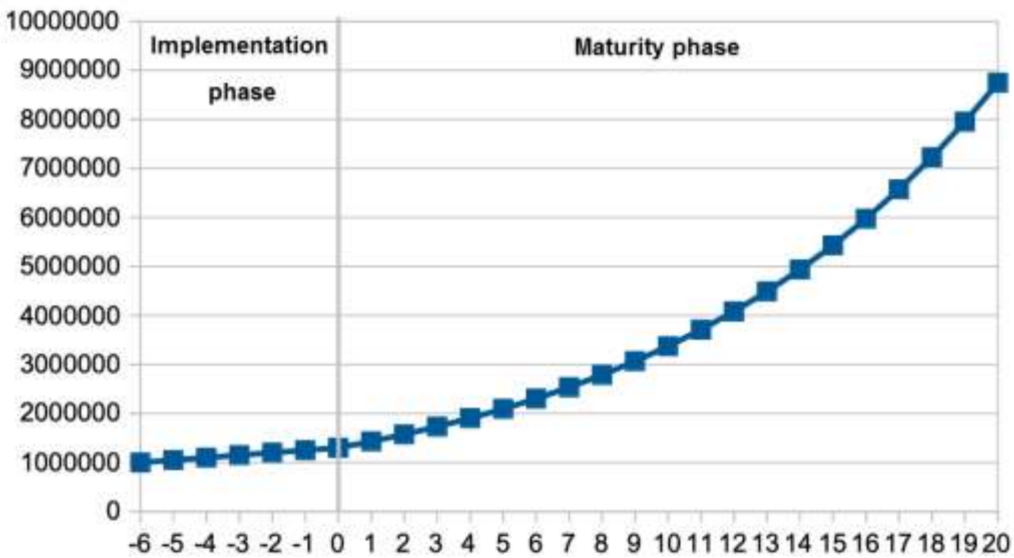

Figure 1. Capital evolution due to the Logarithmic Risk Distribution

An important observation is regarding of axis of the abscissae. This must not be confused with the time axes. By our hypothesis, a time period on the maturity phase is defined by recording the p profit rate. Due to the business characteristics, the time consumed can be different from a period to another. Usually, once the capital is increased, the time spending for each phase can be higher.

Another important observation is regarding safe capital. Even the capital is importantly increased in the maturity phase, the risk is higher from one interval to another. This means the safe capital expressed as a percentage is decreasing once the risk is increased. This is figured in figure 2.

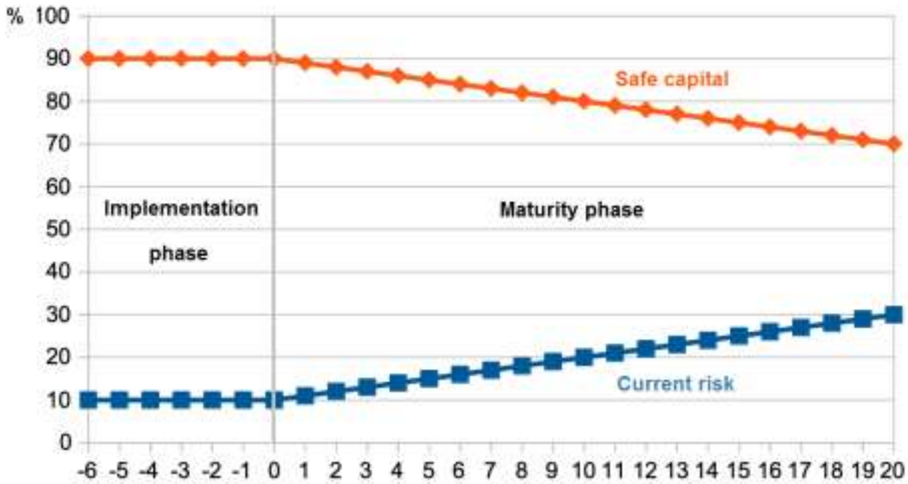

Figure 2. Risk evolution and safe capital evolution

In the example above, it was considered a case in which the risk was increased in the maturity phase, with 1% each time when a 10% profit stake was recorded. This can be a typical case of any stable business in which the losses are quickly recovered and have a lower impact on capital evolution. In these cases, the x-axis can be in a direct correlation tithe the time spent.

A different situation is used in those cases when a loss decreases the involved capital significantly in the current business. In the capital investment activity, after a loss, the capital decreases with that loss. If the risk is high, the loss is important, and the time consumed to recover can be significant. In cases like this, the risk level is lower, and the risk gradient in the maturity phase is also lower. After each loss, the formula (10) will provide the new lower risk level, with some intervals back on the x-axis. The risk will be increased again after some profit stakes will be recorded, following the same risk distribution.

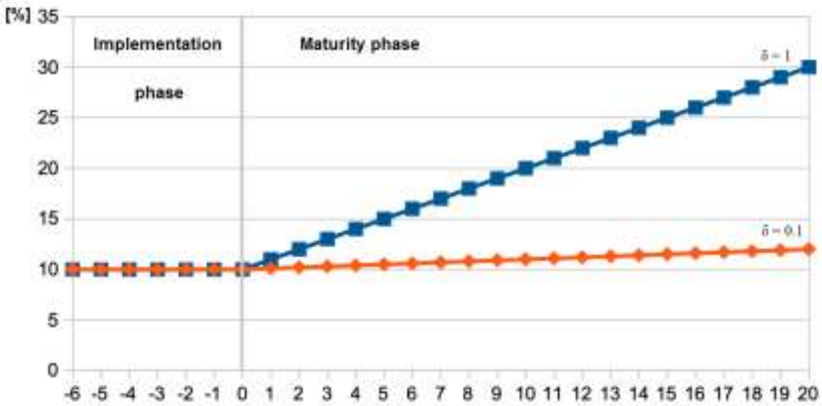

Figure 3. Risk comparison for different risk gradients

For the capital investment and financial trading activities, the Logarithmic Risk Distribution can be successfully combined with the „Global Stop Loss" methodology (Pauna, 2018). Especially for the automatic trading and investment systems, the risk calibration needs a distribution function in order to set the risk automatically depending on the results obtained. The risk distribution presented in this paper is adaptable by the results obtained in each moment of time, can be programmed, is simple, and can be computed in real-time. The Logarithmic Risk Distribution, together with the Global Stop Loss methodology, builds together a dependable risk management procedure for any automated trading software. The steps to be considered in order to implement this methodology package are the next:

a) Set an initial capital (C) together with an initial global risk level (r) and the implementation profit level (p). With all of these, considering that the automated trading system has a positive expectancy strategy, the system will trade the capital until the profit (P) is achieved. In all this period, the risk level is set to be constant at (r) no matter the capital evolution is achieved.

b) Once the implementation phase is ready, and the profit P is collected, the maturity phase can start. For this, the software designers will set the risk gradient δr and the profit gradient δp or the link between these tow factors measured by the ξ coefficient.

c) In every moment of time, when the capital management procedure is called, the available capital will define the risk level using formula (10). If the capital is increased or decreased by the last capital trades, the formula (10) will provide the proper risk calibration according to the money management strategy set by the functional parameters used.

d) Optimizations for the gradient measures δr and δp can be made. The values can depend on the software specificity, on the trading frequency, and, more importantly, on the expectancy and risk to reward ratio assured by the investment capital strategies included in the automated trading software used. This optimization is one of the most important factors in long-term efficiency. A significant number of big losses can decrease the profit ratio drastically. Meanwhile, a small number of loss trades strategy can improve the value for the risk gradient, which can contribute significantly in the capital efficiency on the long-term.

A significant number of capital investors and traders are still using the individual stop-loss protection for each trade. The Logarithmic Risk Distribution can also be used in these cases after each trade. The risk can be set anytime, depending on the current capital level. For this purpose, an automated trading system is a right solution.

The real-estate domain is another example of large investments with high running costs and lower profit rates. The Logarithmic Risk Distribution can be used in order to set the expenditure level for each moment of time, depending on the liquidities. In the risk distribution formula, the capital can be substituted by two factors: the invested capital (CI) and the liquid capital (CL). While the invested capital is almost constant in real estate, the liquid capital is variable and depends on the incomes, on the current expenses, which can be assimilated with a risk. Form this substitution, the current expenditure level can be computed depending on the incomes.

6. CONCLUSIONS

The Logarithmic Risk Distribution is a reliable function which permits to set the risk level depending on the current capital involved in any business. The model was developed by splitting the business scenario into two different phases. The first one is designed for the business implementation period. The business plan is applied until a specific profit level is recorded using a constant risk level. The second phase, called the maturity phase, is the period when the risk is increased with small gradients in order to speed up the business and to increase capital efficiency. The model can be applied both for businesses with the profit ratio higher or lower the risk ratio.

For the maturity phase, the risk is increased with small gradients each time when a small profitability level is obtained. The dependency between the risk level and the available capital is a logarithmic one. This function produces an exponential capital evolution in the maturity phase, the main advantage instead of the linear capital evolution recorded in the implementation phase.

The risk model proposed in this paper can be applied in a large number of economic activities starting with the capital investment, any trading activity, in the real-estate domain, and in any business in which the capital and the risk involved can be measurable. The variables in the Logarithmic Risk Distribution formula are usually included in every business plan, even from the business analysis step. The model has the advantage that is entirely functional for both, the economic activities in which the capital is continuously increasing, but even in those cases in which losses can decrease the capital from time to time. The Logarithmic Risk Distribution will redefine the risk level in the maturity phase for each capital level, assuring a continuous business management activity.

One of the most important advantages of the risk function presented in this paper is simplicity. The Logarithmic Risk Distribution can be computed by anyone for any economic activity using a simple pocket calculator. More than that, the risk function presented can be programmed in advanced business software systems in order to assist the business intelligence procedures in making automated decisions regarding the risk level.

The Logarithmic Risk Distribution is a reliable formula that can be included in any economic and business model which permits to manage a risk increase depending on the available capital.

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