DYNAMIC MODEL OF NEW PRODUCT LAUNCH IMPACT ON STOCK MARKET PARTICIPANTS

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ABSTRACT
The instant access to global information and just as fast responses to the information result in very fast development of communication possibilities and information spreading. The paper addresses spreading information about a new product in a way similar to the principle of virus spreading in epidemiology. We will introduce a mathematical model of signal spreading about a new product in the stock market environment, considering the time delay and inspired by the virus dynamics theory. The mathematical model has been set up from the point of view of the investors´ behaviour and analysed in terms of the time delay influence. The situations demonstrated confirm that the signal of the newly introduced product will have an effect on the stock market behaviour over a certain period but in the long-term perspective its influence disappears. The model solvability is analysed and the solution is demonstrated on a particular example.

KEYWORDS: *dynamic modelling, investment, new product launch, time delay, virus dynamics.*

1. INTRODUCTION

The mathematical models of space-time infectious disease spread help to reveal the causes of the disease origin and most often epidemic spread. Epidemiology employs models allowing the infection spread to be monitored during interactions among people present in a space or social network. A frequent goal is to establish the critical parameter values at which epidemics start spreading in the population. Apart from the usual use, the epidemiological models can also be used to model fundamentally close processes, such as fire spreading, plant invasion in unoccupied spaces, food chain dynamics, etc.
The objective of the activities related to modelling and simulation is not to create a mathematical or computer equivalent of a real system, rather to use the model to understand the behaviour of real systems in order to be able to improve the prediction of and to optimize the behaviour of real objects, or to create new objects.
The paper formulates a model of spread of information about a new product in the stock market, described by a non-linear set of differential equations with delayed argument, inspired by virus dynamics. The model describes the impact of new product launch on investor behaviour provided that the investor response is not immediate and accounts for the time required to make a decision. The paper presents a possibility of non-linear dynamic model solution applying the modern theory of functional differential equations, namely the global gradual approximation method. The application part shows the possible solution on a particular example. A computer simulation demonstrates the model behaviour. The scientific goal is to verify the solvability of the task. Maple was used for graphical result presentation.

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1.1 Literary review

It is nearly impossible to predict e.g. whether a certain product will become a hit, considering the variety of consumer preferences. On the contrary, it can be easily demonstrated that the product spread dynamics may differ significantly for various social link structures (speed, step changes, etc.). Yet new products play an important part in creating and maintaining company competitive advantages. The study (Zou & Li, 2016) investigates the degree of a brand crisis impact on stock prices of competitors within the same product category. Companies capable of repeated new product launches undoubtedly enjoy a more advantageous status in the competitive market environment, in particular if the companies are listed. Every following notification of a new product launch (NPL) is then a matter affecting the vast majority of shareholders (Wang et al., 2011). Investors often base their decisions on whether to buy or sell on a set of many other pieces of information (Aspara et al., 2015). However, a new piece of information about a product may influence the investor not only towards buying but also towards selling, as confirmed by the paper (Ni et al., 2015) analysing the consequences related to a negative stock market reaction to a notification of toy discontinuation. The publication authors (Li & Wang, 2016) have established that a new product launch notification combined with information on research and development investments has a positive effect on investor reaction. However, slight differences in the initial conditions may have serious consequences in the long run. Raassens et al. (2012) state that NPL by a listed company tends to be read as a signal of upcoming stock exchange fluctuations. Presently the potential impact of new product launch notification on the stock exchange behaviour has become the subject of many research papers (Chen et al., 2014), (Lee & Chen, 2009). However, the majority of these studies view the events in the stock exchange in terms of statistical analysis and fail to notice the impact of the new product launch on the stock market.

This process may be influenced by various factors, such as investor sensitivity to information and their mental state, which also applies to other investors, as well as the general stock market environment.

The way people are affected has been interpreted as a process close to the virus infection process in the virus dynamics theory. The spread of a certain type of behaviour is a very similar case (e.g. a spread of a game strategy, political opinion, new technology or a fashionable product, etc.). In this context the scientists have come up with the viral marketing and viral advertising theory (Schullze et al., 2014), (Hayes & King, 2014) with a revolutionary impact on the field of marketing. Social networks, where viral marketing is frequently used, are considered a very effective merchant marketing platform (Faruque et al., 2016). Viral marketing represents a method used for achieving an exponential growth of brand (or product or service) awareness through uncontrolled spread of information among people, this avalanche-like spread being compared to a virus epidemic – hence the method name (Hao & Wang, 2016). López-Fernández (2015) puts viral marketing in a broader context in his book, including a contemplation of corporate responsibility. Arcos et al. (2014) state in their paper that viral marketing may be beneficial in launching a new product as well; however, its effectiveness and campaign measurements may represent a weakness of the strategy.

2. MODEL FORMULATION

The mathematical models of virus spread were originally created in order to estimate the progress of the spread and keep epidemics under control. These models provide answers to a number of questions, e.g. in relation to thresholds and dependence on changes in the respective parameters. They can be applied in planning, comparing and implementation or optimization of various prevention, medical and monitoring programmes. We can come across models for virus population network and immune response comprised of differential equation systems (Browne & Smith, 2018) or (Wang et al., 2018).

Let us use the virus dynamics model presented e.g. in (Nowak & May, 2000) as a basis; however, we shall account for the fact this model does not include the incubation period that elapses between the exposure and the moment when the disease symptoms show. This assumption may be applied when the latent incubation period is relatively short compared to the period of the disease itself. However, if the incubation period is relatively long, the model must be modified by implementing a time delay $\tau(t) > 0$.
Then for $t \geq 0$ the model may be:

$$\frac{dT(t)}{dt} = \lambda - d_1 T(t) + \beta T(t)V(t)$$
$$\frac{dI(t)}{dt} = \beta T(t-\tau(t))V(t-\tau(t)) - d_2 I(t)$$
$$\frac{dV(t)}{dt} = -d_3 V(t) + kI(t) \tag{1}$$

where $T(t), I(t)$ and $V(t)$ mark healthy cells, infected cells and free virus, constant λ represents the level of cell regeneration, d_1, d_2, d_3 mark the death rate of healthy cells, infected cells and free viruses. β is a parameter of the degree of contact between healthy and infected population. $\frac{k}{d_2}$ is the average number of viruses produced by one infected cell. λ, d_1, d_2, d_3 and β are positive constants.
The initial conditions following from the context are defined:

$$T(0) = T_0, I(0) = I_0, V(0) = V_0, \tag{2}$$

where T_0, I_0, V_0 are positive constants.
For $t < 0$ we define

$$T(t) = T_h(t), I(t) = I_h(t), V(0) = V_h(t), \tag{3}$$

where $T_h(t), I_h(t), V_h(t)$ are continuous functions, usually called the "historical function".
The process of "infection" in the stock exchange environment may be relatively complex and depends on many factors, such as sensitivity to information, mental state of the investors, their opinions and behaviour, the stock market environment, etc. To simplify, let us assume the process is analogical to the process of viral marketing and may be divided into several stages, which is also in line with the publications of Hao and Wang (2016) and Conelly et al. (2010).
The NPL signal is sent by the listed company and detected by potential investors, whereby it comes into awareness (Awareness). Then the potential investors contemplate the situation and evaluate whether it is advantageous for them to invest (Evaluation). At the end, the potential investors make a final decision on the investment, execute it and join the shareholders (Investment).
When creating the model, we assume that one investor only invests in one share. The number of potential investors is $I(t)$, the number of shareholders is $I^*(t)$.
The potential investors have a natural source represented by constant λ and the degree of its decrease, i.e. the degree of leaving the stock market, is represented by constant d_1. Not all investors are necessarily aware of the NPL signal, whose intensity we shall mark as $S(t)$, therefore the market sensitivity to information $\beta I(t)S(t)$ must be taken into account as well, where β is a positive constant.
Since the investor needs a certain time lag in order to evaluate the situation, we also consider the effect of the delay of this stage. Provided that the potential investor detected the signal at the moment $t - \tau(t)$, the investor evaluates the signal in the interval $(t - \tau(t), t)$. However, not all potential investors decide to invest the available resources to the share, after having considered all the circumstances. To the contrary, they might sell shares they already own. The investor sentiment

index is considered a decisive factor in the Evaluation stage. (Ljungqvist et al, 2006),(Baker and Wurgler, 2006).
$\beta I\big(t-\tau(t)\big)S\big(t-\tau(t)\big)$ is then the increase in the number of shareholders controlled by the NPL signal and δ marks the degree of natural shareholder number decrease.
The NPL signal itself changes over time not only in relation to advertising and the information spread in the media, but it is also released continuously. The d_3 parameter marks the signal change. Apart from their own judgement, of course, potential investors may be significantly influenced by existing shareholders through interpersonal communication (Hoffman & Broekhuizen, 2009). For this reason, some shareholders may affect the signal intensity. The k parameter represents the degree of contact between shareholders and potential investors.
Thus for $t \geq 0$ the new model may be:

$$\frac{dI(t)}{dt} = \lambda - d_1 I(t) + \beta I(t)S(t),$$
$$\frac{dI^*(t)}{dt} = \beta I(t-\tau(t))S(t-\tau(t)) - d_2 I^*(t),$$
$$\frac{dS(t)}{dt} = -d_3 S(t) + kI(t), \tag{4}$$

where λ, d_1, d_2, d_3 and β are positive constants and we require

$$I(0) = I_0 > 0, I^*(0) = I^*{}_0 > 0, S(0) = S_0 > 0, \tag{5}$$

where $I_0, I^*{}_0, S_0$ are positive constants.
For $t < 0$ we further define that

$$I(t) = I_h(t), I^*(t) = I^*{}_h(t), S(0) = S_h(t), \tag{6}$$

where the historical functions $I_h(t), I^*{}_h(t), S_h(t)$ are continuous.

3. MODEL SOLUTION

3.1 Positive solution

Theorem 1
The solution of the (I, I^*, S) system (4) is positive for $\forall t: t > 0$.
Proof

a) Let us now assume that $I(t)$ is not always positive, therefore there are $t > 0$ for which $I(t) \leq 0$. Since the initial conditions (5) of the system (4) imply that at the start the solution $t = 0$ is positive, there must be the lowest number $t_1 > 0$ that $I(t_1) = 0$. If we substitute this value in the equation $\frac{dI(t)}{dt} = \lambda - d_1 I(t) + \beta I(t)S(t)$ the result is $\frac{dI(t_1)}{dt_1} = \lambda > 0$.
   With regard to the continuity $I(t)$ there is $\eta > 0$ that for $t \in (t_1, t_1 + \eta)$ it is $I(t) < 0$. However, then for t_1 it is $\frac{dI(t_1)}{dt_1} = \lambda \leq 0$, which is disputable.
   Therefore for each $t > 0$ there is $I(t) > 0$.
b) For $I^*(t)$ it may be derived from the equation $\frac{dI^*(t)}{dt} = \beta I(t-\tau(t))S(t-\tau(t)) - d_2 I^*(t)$ that

$$I^*(t) = e^{-d_2 v}\left(I^*(0) + \int_0^u \beta e^{d_2 v} I(v-\tau)S(v-\tau)dv\right) > 0,$$

Thus $I^*(t) > 0$ for each $t > 0$.

c) Let there be such a t_2 that $S(t_2) = 0$. The result after substitution in the equation $\frac{dS(t)}{dt} = -d_3S(t) + kI(t)$ is $\frac{dS(t_2)}{dt} = kI(t_2) > 0$. Analogically to (a), it may be derived that for each $t > 0$ there is $S(t) > 0$.

Thus for system (2) it applies that its solution (I, I^*, S) is positive for $\forall t: t > 0$.

3.2 Solution construction

From a mathematical perspective, system (2) is a non-linear system of three regular first-order differential equations with a non-constant delay.

Functions I, I^*, S are generally unknown functions λ, d_1, d_2, d_3 and β are positive constants and τ functions of delayed impact I^* and S on the solution of system (2).

Therefore system (2) belongs to the (generally non-linear) systems of regular delay differential equations or, more generally – systems of regular differential equations with deviating arguments, or systems of functional differential equations.

As a result of the global nature of the solutions of differential equations and systems with delay and the only recent possibility to start applying general theories of such equations, the methodology of the numerical solution of these equations is now being built. Only the modifications of the local methods of Runge-Kutta are used more widely as well as the step method specific for delay equations – mostly suitable for constant delay equations.

The methodical procedures employed by I. Kiguradze (1988, 1997) and Gelshvilli and Kiguradze (1995) and his colleagues, which will be used for the solution of our problem, are based on the methods of a priori estimates of the solution of such equations and systems and constitute a natural basis of applying the gradual approximation method to a suitable affiliated contractive operator equation.

In analogy to the procedures used in the quoted papers, we will look for a solution of our system (2) as a (direct) limit of task solution sequence ($n \in N$), for $t \geq 0$:

$$\begin{aligned} \frac{dI_n(t)}{dt} &= \lambda - d_1I_n(t) + \beta I_n(t)S_n(t) \\ \frac{dI_n^*(t)}{dt} &= \beta I_{n-1}(t-\tau(t))S_{n-1}(t-\tau(t)) - d_2I_n^*(t) \\ \frac{dS_n(t)}{dt} &= -d_3S_n(t) + kI_n(t) \end{aligned} \tag{7}$$

$$I_0(0) = I_h(0), > 0, I_n^*(0) = I^*{}_h(0) > 0, S_0(0) = S_h(0) > 0 \tag{8}$$

where $I_0(t), I^*{}_0(t), S_0(t)$ are adequately selected (real with regard to the modelled situation) continuous "initial" functions.

At the same time, for $t < 0$

$$I_n(t) = I_h(t), I^*{}_n(t) = I_h^*(t), S_n(t) = S_h(t), \tag{9}$$

where the historical functions $I_h(t), I_h^*(t), S_h(t)$ are continuous.

At the same time, the continuity of the right equation (2) sides and the "initial" functions show definite solvability of tasks (3) for each $n \in N$ and the fulfillment of the definite solvability of the initial task for system (2) shows convergence of the contructed sequence of solutions of tasks (3) to the solution of system (2).

4. NUMERICAL EXPERIMENT

To demonstrate the possibilities of the new approach to the solution of the initial problem, let us assume the "historical development" prior to $t = 0$ may be simulated, for the sake of simplification,

by constant functions $I_h(t) = 250$; $I_h^*(t)$=20; $S_h = 2500$. The calculations were performed using Maple, a multi-purpose mathematical software tool. It provides an advanced, highly effective mathematical computing core with a fully integrated numeric and symbolic.
The constants were selected as follows: $\lambda = 15;\ \mathrm{d}_1 = 0{,}01;\ \mathrm{d}_2 = 0{,}3;\ \mathrm{d}_3 = 2{,}4$
An equilibrium value was determined for the above parameter values and initial conditions. The trivial idle state is $E_1 = (\bar{I}, \bar{I^*}, \bar{S}) = (0{,}1500{,}0)$, which means there is no shareholder and no signal of a new product. For non-trivial state, $E_2 = (\bar{I}, \bar{I^*}, \bar{S}) = (260, 43{,}4769)$.
Furthermore, we created a simulation of the model solution for various lengths of the time delay.
Figure 2 shows that the respective solutions oscillate and finally converge around values corresponding to the values of equilibrium E_2.
Figure 3 presents a case based on the assumption that $I_h(t) = 10$; $I_h^*(t)$=1000; $S_h = 0$, therefore the state is close to E_1. Again, the respective solutions oscillate and converge very quickly to the values of equilibrium E_2. This case clearly shows the oscillation frequency increases with increasing delay.

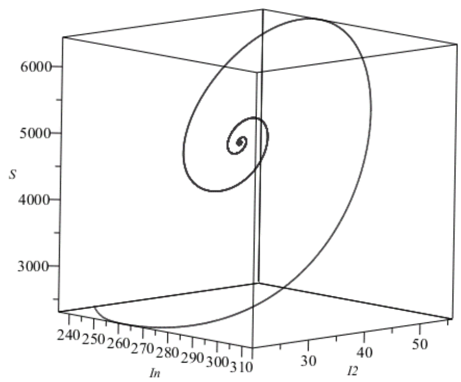

Figure 1. Model solution without delay, i.e. $\tau = 0$;
Source: authors' calculation

5. DISCUSSION

Based on the model we created and the simulations we obtained, potential investors in shares of a company launching a new product (typically an anticipated product, such as the new iPhone) may be divided into four groups: threatened by infection, infected, immune and cured.

- Threatened by infection – potential investors
- Infected – shareholders
- Immune – threatened by infection; however, a portion of them are immune to the infection, in other words, potential investors who do not show interest.
- Recovering (cured) – former shareholders who have already sold their shares

Model investor behaviour expects the "infection" to spread along with the share price increase. Nobody wants to miss an opportunity to make money. The more people hold these shares, the more the portion of potential new buyers (hosts) is reduced and thus the share of people who are potential sellers (infected) increases; at the same time, with a delay, there are those who are already selling (cured). In the long run, this results in price stabilization as well as stable levels of shareholder numbers. However, gradually, as the random shocks divert the ratio in favour of the sellers, the signal strength will decrease and the prices start falling at the same time. Similar dynamics is demonstrated by infectious diseases when they reach a point where a sufficient portion of the people become immune so there are no secondary infections anymore…

The main variable in our model, which affects the share price increase or decrease, is the ratio of potential investors I who know about the share (and its product) and who are willing to invest in it (are susceptible to infection). In analogy to a virus infection progress, we further assume a certain time lag between exposure and infection (obtaining the information about the new product) and infection outbreak (share purchase, i.e. change in the number of shareholders I*), which in our case is caused by the necessity to consider the investment prior to execution. This time lag may also be observed in the model situation graphs (see Figures). The model situations described above confirm that signal S of a newly launched product will affect the stock market behaviour for some time; however, in the long run, its impact is depleted, the market stabilizes and the level of shareholder and potential investor numbers will stabilize at the equilibrium.

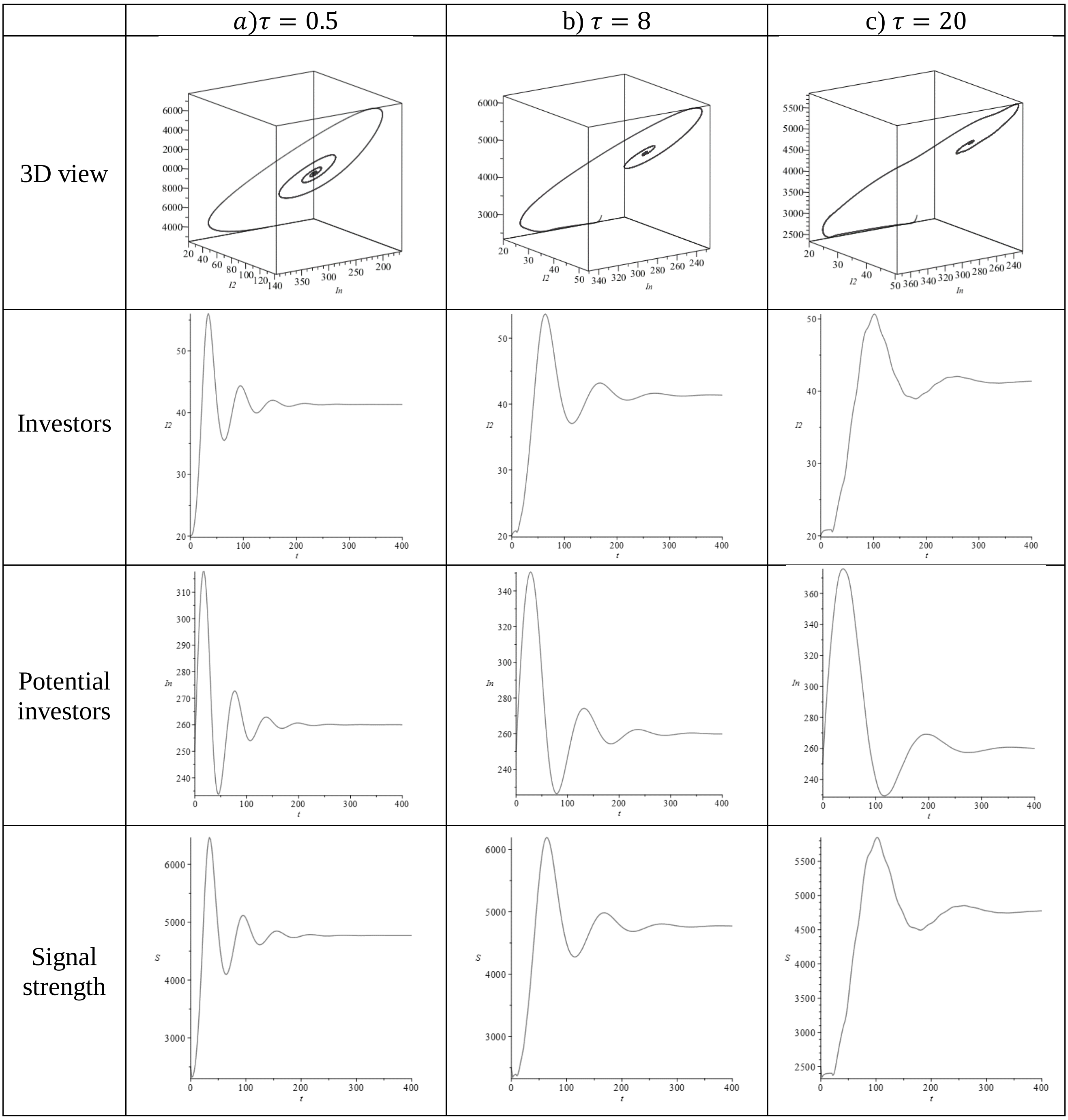

Figure 2. Model simulation for various delay lengths

Source: authors' calculation

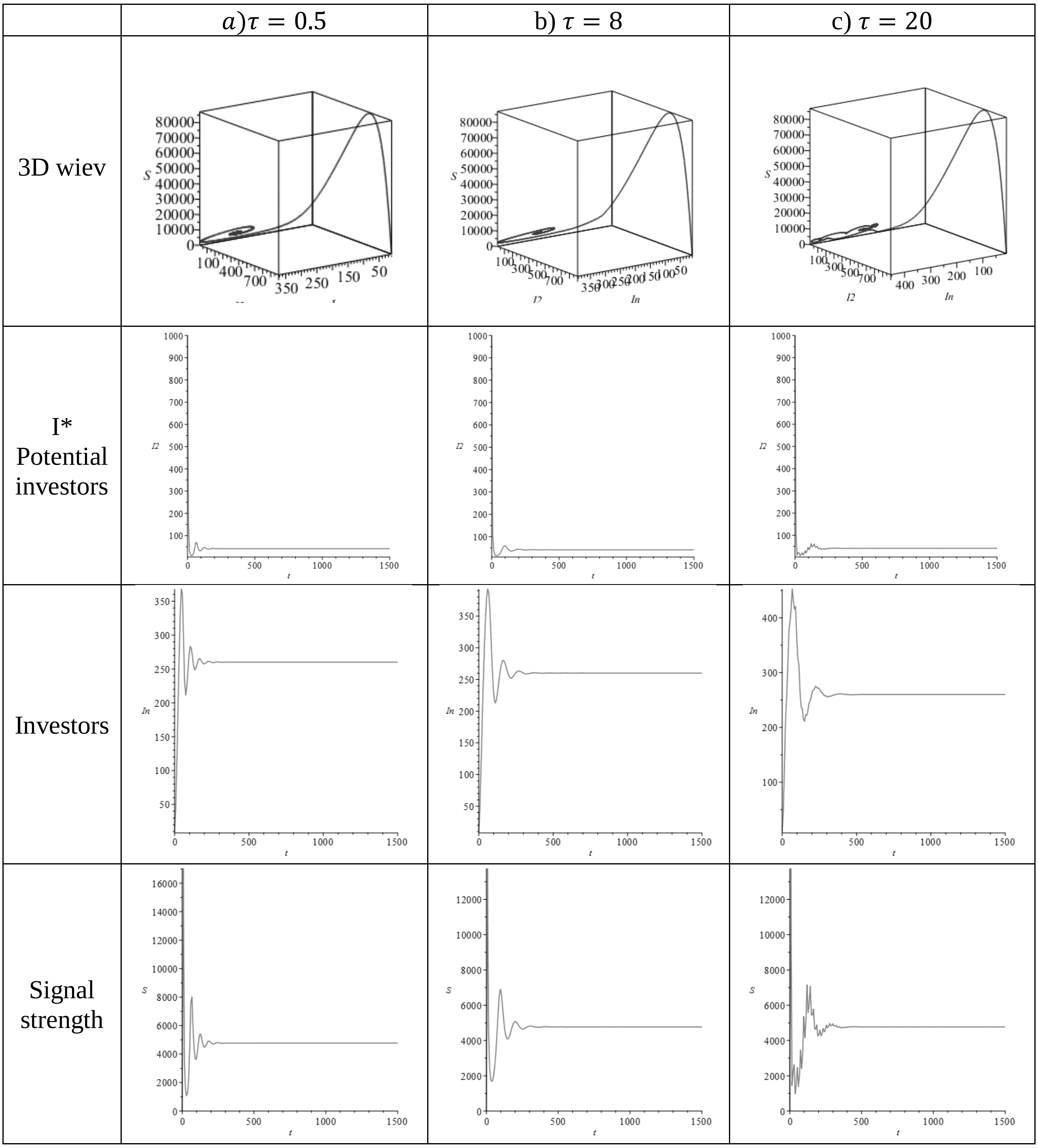

Figure 3. Model simulation for various delay lengths for the start close to1

Source: authors' calculation

6. CONCLUSION

For enterprises, launching new products is a key moment of their existence. Companies that do not invest in innovation and new product development expose themselves to a great danger of their products no longer satisfying the demands of the customers in the environment of ever-changing technical and technological procedures and customer requirements, resulting in the customers moving to competitive products. In this paper, we created a mathematical model of spreading a new product signal in the stock market conditions, accounting for time delay and inspired by the viral dynamics theory. The mathematical model has been created from the investor behaviour perspective and analysed in terms of time delay impact caused by the time needed by a potential investor to make a decision.

Our model is also in line with the sentiment theory, which claims that a large wave of positive sentiment has a disproportionate impact on the share price increase, the share price evaluation being high as it is, and if the expected large profit is not realized (e.g. from the new product), investors leave…

The application of this model using real data may be very useful in reducing the risk for potential investors since there is nothing worse for an investor than to buy a share in a company that was growing (often rapidly) at the peak and then to watch the company share price decrease or even plunge, although the company reports increasing sales of the new product. Beware though, the investor should realize the number of those who bought some time ago and now are selling (i.e. recovering/cured sellers) may be increasing.

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