STOCK INVESTMENT MANAGEMENT UNDER UNCERTAINTY

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ABSTRACT
This paper presents a stock investment management problem under uncertainty solved by applying a portfolio selection algorithm for interval attributes. The current top 10 most traded stocks on the Bucharest Stock Exchange Market were taken into consideration in this study. The results indicated that according to the decision maker risk attitude there are several different final portfolio structures. For instance, for the case of a risk neutral decision maker, the structure was similar to the case of a risk adverse decision maker, while in the case of a risk lover decision maker, however, the final stock portfolio had a more different structure.

KEYWORDS: *interval data, stock portfolio selection, uncertainty*

JEL CLASSIFICATION: *G11, D81*

1. INTRODUCTION

Stock markets are turning into very unstable investment fields under economic recession and financial instability. In this context stock investment management should turn into a true business challenge for any investor. That is why new and efficient algorithms are expected in order to solve complex decision problems.
The literature review is quite vast and offers various efficient decision making techniques under complete information (see Resteanu et al., 2007), under risk (see Andreica et al., 2008), or under uncertainty (Amiri et al., 2008; Andreica et al., 2010; Chen et al., 2009; Ye and Li, 2009).
One of the best known decision making optimization methods are multi-attribute and multi-objective decision making, fuzzy decision rules (Stoica et al., 2008) and dynamic programming.
According to Andreica et al. (2010) multi-attribute decision making (MADM) refers to making preference decisions over the available alternatives that are characterized by multiple, generally conflicting attributes. In classic MADM problems, most of the input variables are assumed to be crisp data. However, since in most cases it is quite difficult to precisely determine the exact value of the attributes under incomplete information and uncertainty, their values are better described using interval data.
In this paper we solve a stock portfolio selection problem under uncertainty by applying a portfolio selection algorithm for interval data attributes. The paper is structured as follows. In section 2 we summarize the portfolio selection method for interval data proposed by Andreica et al., 2010, while a stock portfolio decision problem will be solved in Section 3. Section 4 concludes.

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2. THE PORTFOLIO SELECTION ALGORITHM FOR INTERVAL DATA

In this section we summarize the extension of a portfolio selection algorithm previously proposed by Andreica et al. in 2010 for the case of interval data. The portfolio selection problem consists in how to allocate an amount of money to a number of goods or stocks in order to bring a most profitable return for investors. Therefore, the algorithm implies using both a TOPSIS and an ELECTRE III method for interval data in order to select the best portfolio structure. The reason for choosing these 2 particular methods consists in the fact that the TOPSIS method is an effective method to determine the ranking of decision alternatives, but cannot, however, distinguish the difference degree between two decision alternatives easily. On the other hand, although the ELECTRE III method can easily compare the degree of difference among all alternatives, it cannot always provide total ordering. That is why, when combining the 2 methods, a final portfolio with improved characteristics is obtained.

The general MADM problem is presented in the form of a matrix, in which there are *m* rows, representing different alternatives and *n* columns, representing the criteria specifying the properties of the alternatives. In the interval data approach the assessment of alternative A_i with respect to criterion C_j is represented by the intervals $[x_{ij}^L, x_{ij}^U]$ and the vector of weights is replaced with a vector whose components are intervals $w = \{[w_1^L, w_1^U],[w_2^L, w_2^U],...,[w_n^L, w_n^U]\}$. The decision matrix is as follows:

$$\begin{array}{cccccc} & C_1 & C_2 & \cdots & C_n \\ A_1 & [x_{11}^L, x_{11}^U] & [x_{12}^L, x_{12}^U] & \cdots & [x_{1n}^L, x_{1n}^U] \\ A_2 & [x_{21}^L, x_{21}^U] & [x_{22}^L, x_{22}^U] & \cdots & [x_{2n}^L, x_{2n}^U] \\ A_m & [x_{m1}^L, x_{m1}^U] & [x_{m2}^L, x_{m2}^U] & \cdots & [x_{mn}^L, x_{mn}^U] \end{array}$$

2.1. The TOPSIS method for interval data

The basic concept of TOPSIS method is that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. As presented in Jahanshahloo et al. (2006) the extended TOPSIS method for interval data has the following steps:

Step 1. First calculate the normalized decision matrix, using the formulas for each interval:

$$n_{ij}^L = \frac{x_{ij}^L}{\sqrt{\sum_{i=1}^{m}[(x_{ij}^L)^2 + (x_{ij}^U)^2]}}, i = 1,..m; j = 1,..n \qquad n_{ij}^U = \frac{x_{ij}^U}{\sqrt{\sum_{i=1}^{m}[(x_{ij}^L)^2 + (x_{ij}^U)^2]}}, i = 1,..m; j = 1,..n$$

Step 2. Applying an Imprecise Shannon's Entropy method (see Lotfi et al., 2010) in order to determine the average objective crisp weights $\overline{w}_j$ for each criterion as: $\overline{w}_j = (w_j^L + w_j^U)/2, \quad j = 1,..n$. The weights are then normalize as: $w_j = \frac{\overline{w}_j}{\sum_{j=1}^{n}\overline{w}_j}, \quad j = 1,..n$ so that $\sum_{j=1}^{n} w_j = 1$. After that, the weighted normalized interval decision matrix is constructed as:

$$v_{ij}^L = w_j n_{ij}^L, i = 1,..m; \quad j = 1,..n$$
$$v_{ij}^U = w_j n_{ij}^U, i = 1,..m; \quad j = 1,..n$$

Step 3. The positive and the negative ideal solutions are determined as:

$$A^+ = \{(\max_i v_{ij}^U \mid j \in B), (\min_i v_{ij}^L \mid j \in C)\}$$

$$A^- = \{(\min_i v_{ij}^L \mid j \in B), (\max_i v_{ij}^U \mid j \in C)\}$$

where B is associated with the benefit criteria and C with the cost criteria.

Step 4. The separation of each alternative from the positive and the negative ideal solutions are calculated as:

$$d_i^+ = \{\sum_{j\in B}(v_{ij}^L - v_{ij}^+)^2 + \sum_{j\in C}(v_{ij}^U - v_{ij}^+)^2\}^{\frac{1}{2}}, i = 1,..m$$

$$d_i^- = \{\sum_{j\in B}(v_{ij}^U - v_{ij}^-)^2 + \sum_{j\in C}(v_{ij}^L - v_{ij}^-)^2\}^{\frac{1}{2}}, i = 1,..m$$

Step 5. A closeness coefficient is defined to determine the ranking order of all alternatives in a descending order: $\mathrm{CC_i} = \dfrac{d_i^-}{d_i^+ + d_i^-}, \quad i = 1,..m$.

2.2 ELECTRE III method for interval data

The ELECTRE III method deals with pseudo-criteria instead of true criteria, allowing the following types of preferences between alternatives: strong preference, weak preference and indifference. In order to do that, it uses a preference threshold *p*, an indifference threshold *q* and a veto threshold *v*. The ELECTRE III method extension for interval data presented by Andreica et al. (2010) implies the following steps:

Step 1. A risk attitude factor for the decision maker is first introduced, similarly to (Ye and Li, 2009), in order to transform an interval value into an exact value. In case of a benefit criterion, the exact value x_{ij} is obtained as: $x_{ij} = \bar{x}_{ij} + \varepsilon \cdot \hat{x}_{ij}$, where $\bar{x}_{ij}$ is the middle value of the interval and $\hat{x}_{ij}$ is the width of the interval, measured as: $\hat{x}_{ij} = x_{ij}^U - x_{ij}^L$, while in case of a cost criterion, the exact value x_{ij} is obtained as: $x_{ij} = \bar{x}_{ij} - \varepsilon \cdot \hat{x}_{ij}$. The risk factor ε represents the risk attitude of the decision maker and takes values between -0.5 and 0.5. If the decision maker is risk adverse, then the range of the risk factor ε is $-0.5 \le \varepsilon < 0$, while if the decision maker is risk lover, the risk factor ε is $0 < \varepsilon \le 0.5$. The case in which the decision maker is risk neutral implies that $\varepsilon = 0$.

Step 2. The concordance index $C_j(X_i, X_l)$ is calculated for each X_i and X_l with respect to each criterion j as:

$$C_j(X_i, X_l) = \begin{cases} 1, & X_{lj} - X_{ij} \le q_j \\ \dfrac{X_{ij} - X_{lj} + p_j}{p_j - q_j}, & q_j < X_{lj} - X_{ij} < p_j \\ 0, & X_{lj} - X_{ij} \ge p_j \end{cases}$$

Step 3. The discordance index $D_j(X_i, X_l)$ is then calculated for each pair of alternatives with respect to each criterion:

$$D_j(X_i, X_l) = \begin{cases} 1, & X_{lj} - X_{ij} \ge v_j \\ \dfrac{X_{lj} - X_{ij} - p_j}{v_j - p_j}, & p_j < X_{lj} - X_{ij} < v_j \\ 0, & X_{lj} - X_{ij} \le p_j \end{cases}$$

Step 4. The overall concordance index for each X_i and X_l is determined as:

$$C(X_i, X_l) = \sum_{j=1}^{n} w_j \cdot C_j(X_i, X_l)$$

Step 5. The credibility matrix $S(X_i, X_l)$ of each pair of alternatives is calculated as:

$$S(X_i, X_l) = \begin{cases} C(X_i, X_l), & D_j(X_i, X_l) \le C(X_i, X_l), \forall j \\ C(X_i, X_l) \prod_{j \in SC(X_i, X_l)} \frac{1 - D_j(X_i, X_l)}{1 - C(X_i, X_l)}, otherwise \end{cases}$$

where $SC(X_i, X_j)$ is the set of criteria for which: $D_j(X_i, X_l) > C(X_i, X_l)$

Step 6. Then the concordance credibility and discordance credibility degrees are defined as:

$$\phi^+(X_i) = \sum_{i \ne l} S(X_i, X_l), \quad i,l = 1,..m$$
,
$$\phi^-(X_i) = \sum_{i \ne l} S(X_l, X_i), \quad i,l = 1,..m$$

where the concordance credibility degree represents that the degree of the alternative X_i is at least as good as all the other alternatives, while the discordance credibility degree represents that the degree of all the other alternatives is at least as good as the alternative X_i. Based on these two indicators, a net credibility degree for each alternative X_i is defined as: $\phi(X_i) = \phi^+(X_i) - \phi^-(X_i)$ which has higher values when the alternative X_i is considered more attractive in comparison to the other alternatives.

Step 7. Finally, an outranking index OTI_i, is defined for each alternative in the following manner:

$$OTI(X_i) = \frac{\frac{\phi(X_i)}{m-1} + 1}{2}$$

Based on the outranking index the final ordering of the alternatives is obtained.

2.3 Portfolio selection algorithm

The portfolio selection method under uncertainty proposed by Andreica et al. (2010) requires a combination of the results obtained from the two extended versions of TOPSIS and ELECTRE III methods for interval data. It can actually be assumed that TOPSIS and ELECTRE III methods represent two decision makers of the portfolio selection. That is why the best decision will be made when taking into consideration both experts opinions regarding the set of alternatives that may lead to best portfolio. The portfolio selection algorithm for interval data has the following steps:

Step 1. Apply the extended TOPSIS method for interval data and identify the closeness coefficient CC_i for each alternative.

Step 2. Determine the threshold $\beta_{TOPSIS} = \frac{\sum_{i=1}^{m} CC_i}{m}$ and then identify the investment portfolio set of the TOPSIS method as: $\Omega_T = \{X_i \mid CC_i \ge \beta_{TOPSIS}\}$.

Step 3. Apply the extended ELECTRE III method for interval data and identify the outranking index OTI_i for each alternative.

Step 4. Determine the threshold $\beta_{ELECTRE} = \frac{\sum_{i=1}^{m} OTI(X_i)}{m}$ and then identify the investment portfolio set of the ELECTRE III method as: $\Omega_E = \{X_i \mid OTI(X_i) \ge \beta_{ELECTRE}\}$

Step 5. The decision upon the final investment portfolio set implies the intersection of the two portfolio sets that resulted based on the extended TOPSIS and ELECTRE III methods for interval data, $\Omega_P = \Omega_T \cap \Omega_E$. According to the closeness coefficient, the investment portfolio ratios for the TOPSIS problem are calculated as:

$$P_{T\_P(X_i)} = \begin{cases} \dfrac{CC(X_i)}{\sum\limits_{X_i \in \Omega_P} CC(X_i)}, & X_i \in \Omega_P \\ 0, & otherwise \end{cases}$$

While according to the outranking index, the investment portfolio ratios for the ELECTRE III problem are calculated as:

$$P_{E\_P(X_i)} = \begin{cases} \dfrac{OTI(X_i)}{\sum\limits_{X_i \in \Omega_P} OTI(X_i)}, & X_i \in \Omega_P \\ 0, & otherwise \end{cases}$$

Step 6. Finally, the risk attitude of the decision maker is taken into consideration by assuming that the decision maker can either be risk adverse, risk neutral or risk lover. According to the risk attitude of the decision maker, the final portfolio ratios of the strict investment portfolio set are determined based on the formulas:

$$P_{RA} = \frac{\min(P_{T\_P}(X_i), P_{E\_P}(X_i))}{\sum\limits_{X_i \in \Omega_P} \min(P_{T\_P}(X_i), P_{E\_P}(X_i))}, \qquad P_{RL} = \frac{\max(P_{T\_P}(X_i), P_{E\_P}(X_i))}{\sum\limits_{X_i \in \Omega_P} \max(P_{T\_P}(X_i), P_{E\_P}(X_i))}$$

$$P_{RN} = \frac{\dfrac{(P_{T\_P}(X_i) + P_{E\_P}(X_i))}{2}}{\sum\limits_{X_i \in \Omega_P} \dfrac{(P_{T\_P}(X_i) + P_{E\_P}(X_i))}{2}}$$

where P_{RA} represents the final portfolio ratios in case the decision maker is risk adverse, P_{RN} for the risk neutral case, while P_{RL} represents the final portfolio ratios in case the decision maker is risk lover.

3. THE STOCK INVESTMENT MANAGEMENT PROBLEM

A decision maker wants to invest a sum of money into the Bucharest Stock Exchange Market and takes into consideration the top 10 most traded stocks on the Romanian capital market: TLV, FP, SNP, TGN, TEL, BIO, BRK, BRD, BVB and ELMA, for which monthly risk and return for the period January 2012 – September 2012 are known. In order to calculate each stock's risk and return for these months of the year 2012, we first collected data regarding each stock price evolution from the Kmarket website. We then identified the minimum and the maximum value for each stock risk and return and summarized the results in Table 1.

We considered that both decision criteria concerning risk and return are of equal importance in the portfolio decision problem. The interval decision matrix for the stock portfolio selection problem is presented in Table 1.

The portfolio algorithm for interval data was then applied. First, the TOPSIS method for interval data was used in order to determine the portfolio set of stocks that are the closest to the ideal positive solution and the farthest to the ideal negative solution. It resulted the following complete order of stocks: **FP** (0.601), **SNP** (0.600), **BIO** (0.581), **BRK** (0.559), **TEL** (0.518), **BRD** (0.518), **BVB** (0.499), **TLV** (0.494), **TGN** (0.484) and **ELMA** (0.476) out of which only the first 4 stocks were selected to be the best by having higher closeness coefficient than the average value β_{TOPSIS} of 0,533. The TOPSIS stock portfolio structure is the following: **FP** (25.66%), **SNP** (25.65%), **BIO** (24.82%) and **BRK** (23.87%).

Table 1. The interval decision matrix

	RETURN (max)	RISK (min)
TLV	[-1.16% , 0.73%]	[1.40%, 2.81%]
FP	[-0.55%, 0.88%]	[0.88%, 2.04%]
SNP	[-0.62%,1.00%]	[0.70%, 2.24%]
TGN	[-1.03%, 0.40%]	[0.74% , 2.91%]
TEL	[-0.82% , 0.34%]	[0.71%, 2.01%]
BIO	[-0.38%, 0.55%]	[0.53%, 2,77%]
BRK	[-0.46%, 0.65%]	[1.09%, 3.13%]
BRD	[-0.7%, 0.3%]	[0.7%, 2.6%]
BVB	[-1.4%, 0.9%]	[0.9%, 2.5%]
ELMA	[-1.1%, 0.6%]	[1.3%, 3.5%]

Source: made by authors using data from www.kmarket.ro and www.bvb.ro

Then, the extended version of ELECTRE III method for interval data was applied, in order to identify the second stock portfolio set, based on pair comparisons of each combination of stocks. For that, we first had to decide upon the level of the parameter ε . We used -0.5 in case the decision maker is risk adverse, 0 for the risk neutral case and 0.5 in case the decision maker is risk lover.
Secondly, the threshold levels of the parameters p, q and v were predetermined. Similar to Chen and Hung's approach (2009) in which q= 1/6; p=2/6 and v=3/6, we used the following formulas for computing the thresholds. Let MD_j be the maximum difference between two alternatives for criterion j. We set the indifference threshold q_j to 1/6 * MD_j, the preference threshold p_j to be 2/6 * MD_j and the veto threshold v_j to 3/6 * MD_j.
After that we computed the concordance and discordance index in order to determine the credibility matrix and the concordance and discordance credibility degrees. Based on that, we were able to calculate the OTI values for each alternative and to establish the final ranking of the alternatives for each possible decision maker's risk attitude, as presented in Table 2, where between brackets are the OTI values higher than $\beta_{ELECTRE}$ =0.5.

Table 2. The results of ELECTRE III method

RANK	RISK ADVERSE	RISK NEUTRAL	RISK LOVER
1	**FP** (0.87)	**SNP** (0.90)	**SNP** (0.90)
2	**SNP** (0.81)	**FP** (0.89)	**FP** (0.82)
3	**TEL** (0.78)	**BIO** (0.84)	**BVB** (0.81)
4	**BIO** (0.67)	**BRK** (0.55)	**BIO** (0.64)
5	**BRD** (0.61)	**TEL** (0.55)	
6	**BRK** (0.52)		
	beta=0,5	*beta=0,5*	*beta=0,5*

Source: made by authors using data from www.kmarket.ro

The final portfolio structures when considering the three types of decision maker's risk attitude are described in Figure 1. One can notice that there are several differences in the final ranking of the alternatives based on the decision maker risk attitude, when intersecting the portfolio sets of the two MADM methods.

For instance, in the case of risk adverse decision maker, the best combination of stock investment consists in 27.7% SNP stocks, 27.7% FP stocks, 25.1% BIO stocks and 19.5% BRK stocks.

In case of risk neutral decision maker, the structure is similar to the case of risk adverse decision maker, but one should invest 0.8% less in SNP stocks as well as 0.8% less in FP stocks, 0.5% more in BIO stocks and 1.1% more in BRK stocks.

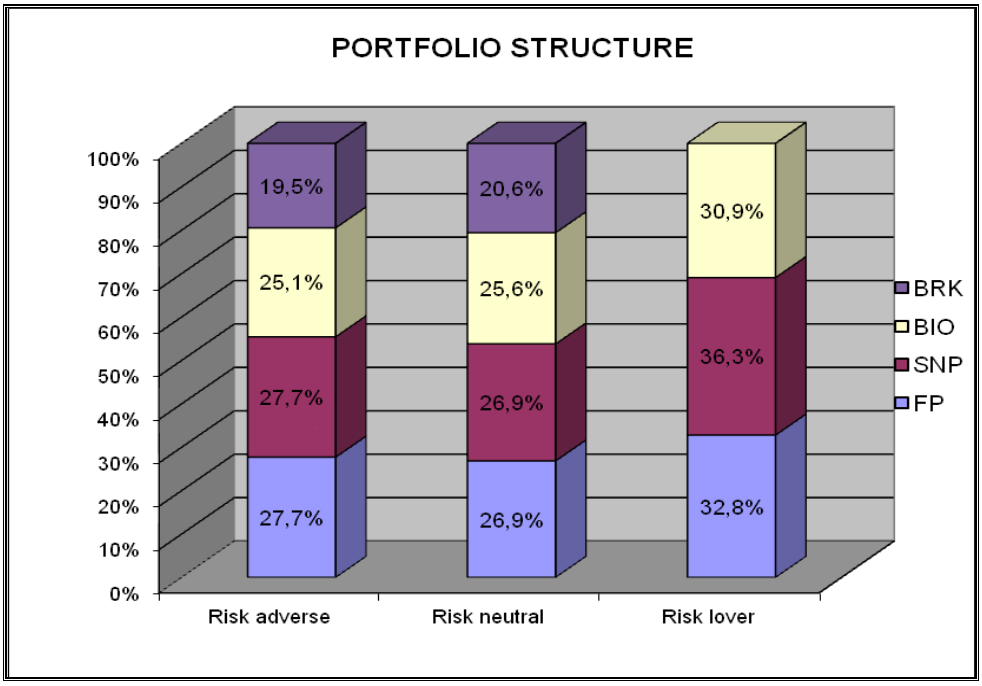

Figure1. The final stock portfolio structure
Source: made by the authors

In the case of a risk lover decision maker, however, the final stock portfolio is made up of only three stocks as comparative to the previous results and has the following structure: SNP in a proportion of 36.3%, FP in a proportion of 32.8% and BIO in a proportion of 30.9%.

4. CONCLUSIONS

In this paper we solve a stock portfolio selection problem under uncertainty by applying a portfolio selection algorithm for interval data attributes, such as: risk and return. The particular portfolio selection algorithm that was applied in this paper has the advantage of allowing describing uncertainty based on interval data attributes.

The results indicated different final portfolio structures according to the decision maker risk attitude. For instance, for the case of a risk neutral decision maker, the structure was similar to the case of a risk adverse decision maker, while in the case of a risk lover decision maker, however, the final stock portfolio had a more different structure than the previous results, indicating that the BRK stocks are less favorable in the final stock portfolio of a risk lover decision maker, since it brings lower returns.

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